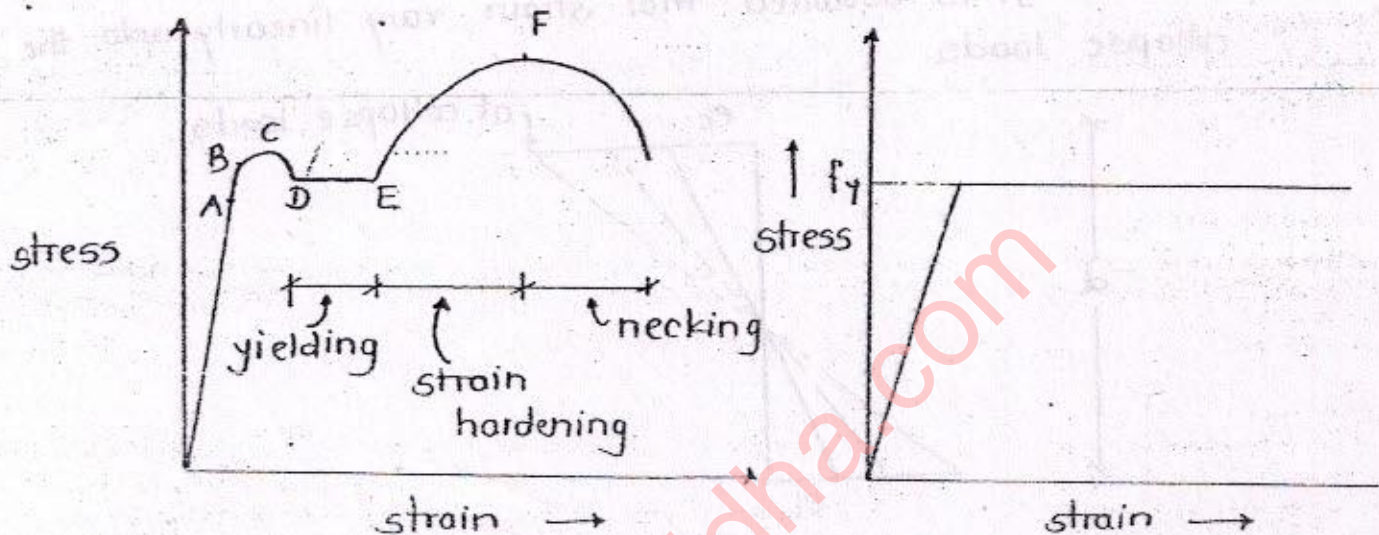


# DESIGN OF STEEL STRUCTURE

Thursday  
5<sup>th</sup> October 2015

Plastic analysis of beams:



Actual stress-strain curve

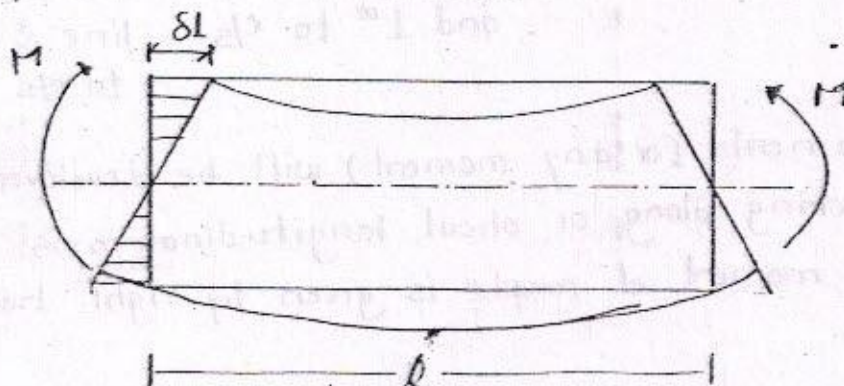
Idealised strain-stress curve.

Plastic analysis of beams is based on Idealised stress-strain curve (the effect of strain hardening is neglected).

Assumptions in plastic analysis:

- (i) Plain section remains plain after bending also. (This assumption is called 'Bernoulli's assumption')

It implies that only strains vary linearly over the depth of the cross section.



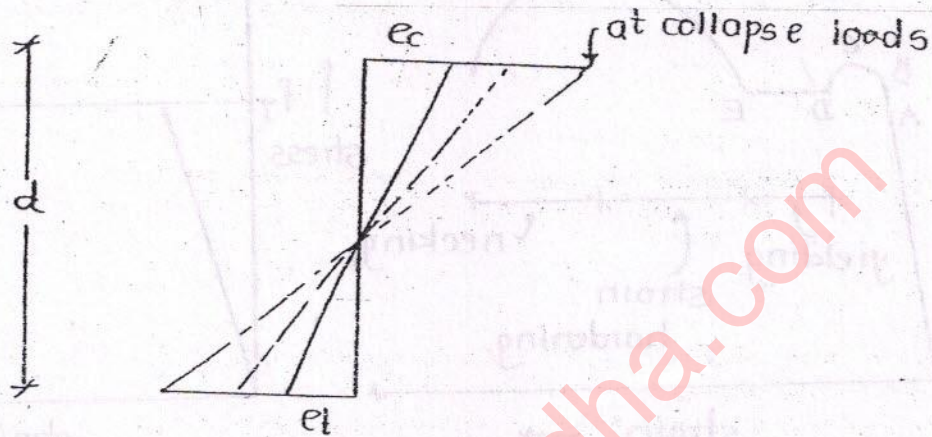


We know that,

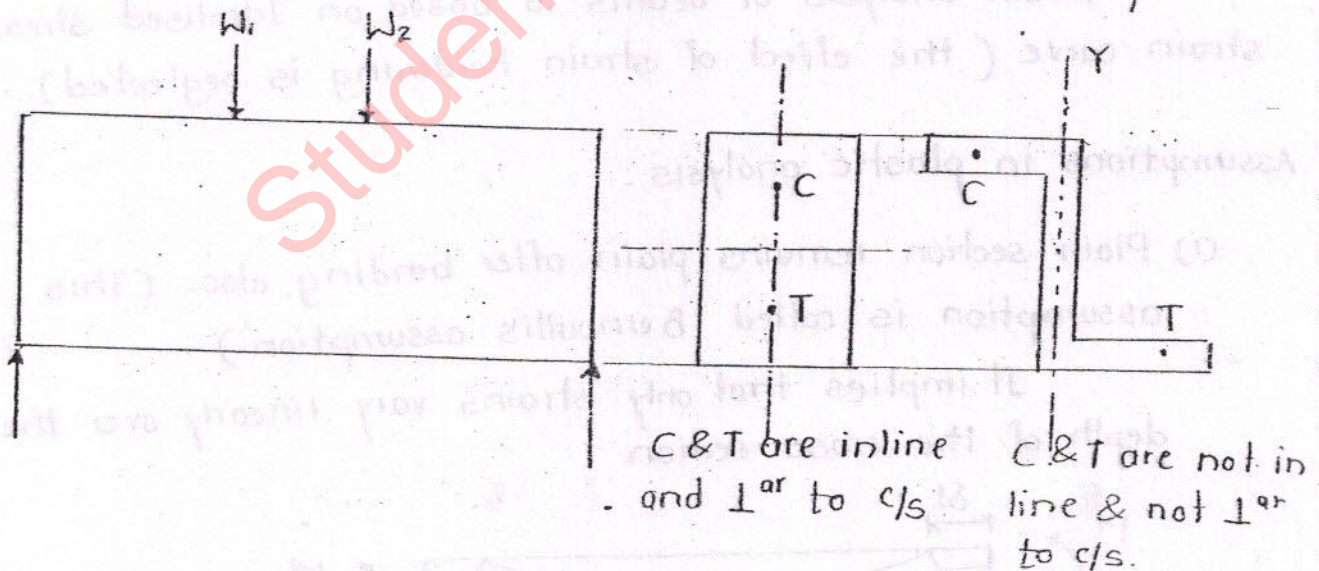
$$\text{strain, } e = \epsilon = \frac{\delta l}{l}$$

As length of beam  $l$  is constant strain ( $e$ ) varies as  $\delta l$  varies. Therefore, the assumption.

It is assumed that strain vary linearly upto the collapse loads.



(ii) The cross section must be symmetrical w.r.t. the plane of loading. Otherwise twisting moments are developed in the beam and flexure formula cannot be applied directly.



Twisting moments (or any moment) will be developed if the moment is acting along or about longitudinal axis.

Direction of moment of couple is given by right hand screw rule.

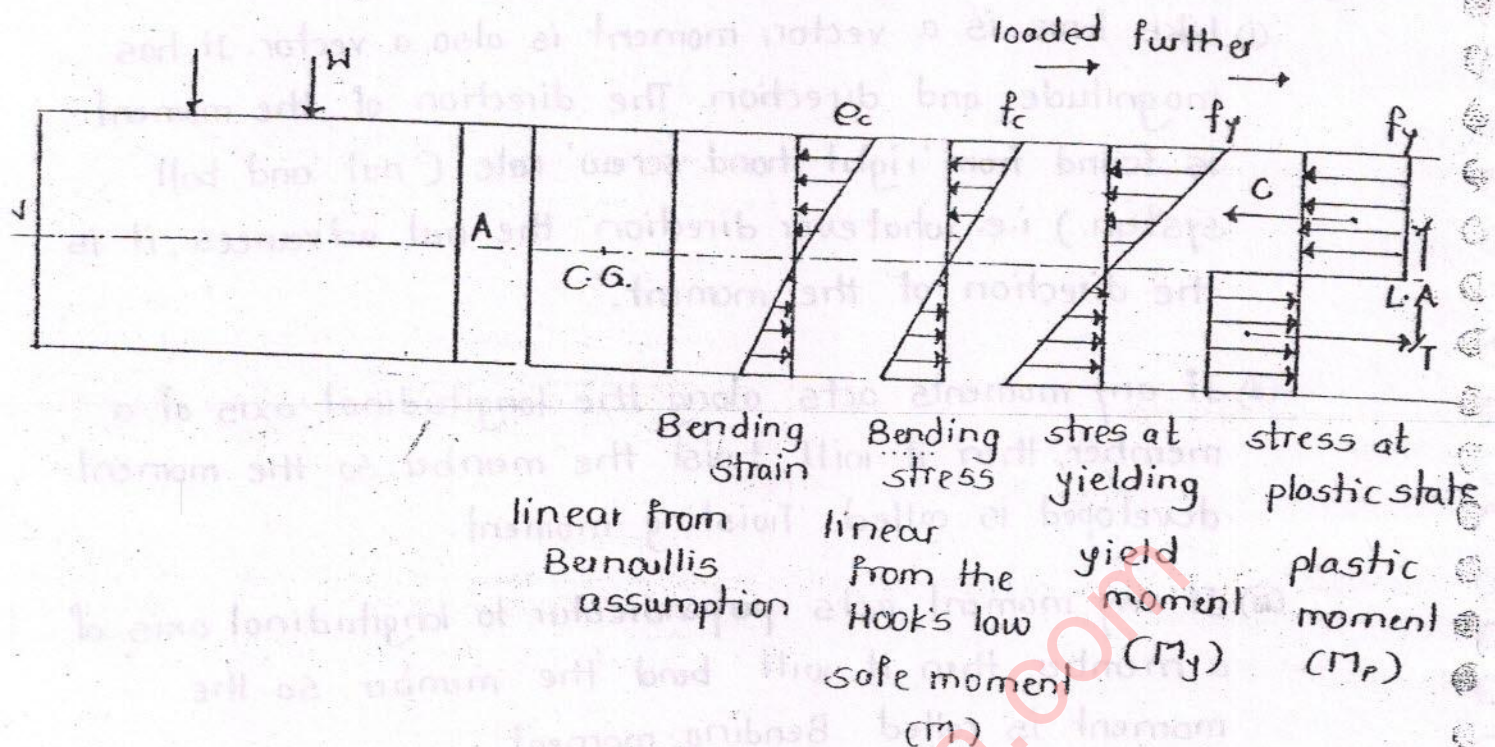


Note:

- (i) Like force is a vector, moment is also a vector. It has magnitude and direction. The direction of the moment is found from 'right hand screw' rule (nut and bolt system.) i.e. whatever direction the nut advances, it is the direction of the moment.
- (ii) If any moment acts along the longitudinal axis of a member, then it will twist the member. So the moment developed is called Twisting moment.
- (iii) If any moment acts perpendicular to longitudinal axis of a member then it will bend the member. So the moment is called Bending moment.
- (iv) A moment can be bending moment or twisting moment depending upon its direction.
- (v) Axial loads and shear force are neglected. i.e. axial deformations and shear deformations are neglected in plastic analysis.
- (vi) Young's modulus  $E$  is same in tension and compression.
- (vii) The stress-strain curve is assumed to be bi-linear. (i.e. it consists of two straight lines)



Plastic moment of a section:



Note:

(i) Safe moment or working moment

$$M = f \cdot Z$$

where,

$Z$  - section modulus, also called as flexural strength parameter.

$$Z = \frac{I}{y}$$

It is called flexural strength parameter because for a given material  $Z$  decides the flexural strength of the beam.

(ii) If the stress variation is linear or triangular distribution we can use  $M = f \cdot Z$  to find moment of resistance.

(ii) Yield moment

$$M_y = f_y \cdot Z$$

We can write this because stress variation is triangular.

$$Z = \frac{I}{y} = \frac{bd^3}{6}$$



(i) Plastic moment:

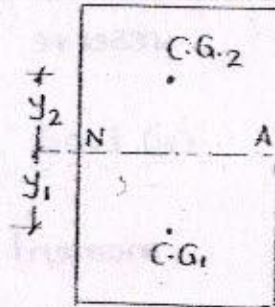
$$M_p = C \times \text{Lever arm} \quad \text{or} \\ = T \times \text{Lever arm}$$

We could not write  $M_p = f_y \cdot Z$  because stress variation is not linear at plastic stage.

$\therefore$  Beam remains in position.  $\sum F_x = 0$

$$\therefore C = T$$

$$M_p = \left( f_y \cdot \frac{A}{2} \right) \cdot (y_1 + y_2) \\ = f_y \cdot \left[ \frac{A}{2} (y_1 + y_2) \right] \\ = f_y \cdot Z_p$$



where,

$Z_p$  - plastic modulus

$$= \frac{A}{2} (y_1 + y_2)$$

$y_1, y_2$  are distances of tension area and compression area from N.A.

(iv) At fully plastic state, N.A. cuts the entire area into the two equal areas. (because from the equilibrium consideration,  $\sum x = 0$ )

Since,  $f_y$  is same throughout c/s, areas must be same.

$$C = T$$

$$\left( f_y \times \frac{A}{2} \right) = \left( f_y \times \frac{A}{2} \right)$$

(v) The ratio of plastic moment and yield moment is called shape factor.

$$\text{shape factor} = \frac{\text{plastic moment}}{\text{yield moment}}$$



$$S.F. = \frac{M_p}{M_y}$$

$$= \frac{f_y \cdot Z_p}{f_y \cdot I}$$

$$\text{shape factor} = \frac{Z_p}{Z}$$

Shape factor represents the reserve strength of the beam section beyond yielding. More shape factor implies more reserve strength beyond yielding.

(vi) Load factor:

It is the ratio of plastic moment and safe moment.

$$\text{Load factor} = \frac{\text{Ultimate load} \xrightarrow{\text{R.C.C.}}}{\text{working load}} = \frac{\text{plastic moment} \xrightarrow{\text{Steel}}}{\text{working / safe moment}}$$

$$= \frac{f_y \cdot Z_p}{f \cdot Z}$$

$$= \left( \frac{f_y}{f} \right) \cdot \left( \frac{Z_p}{Z} \right)$$

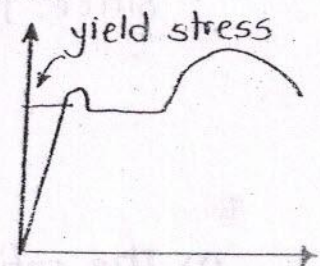
Load factor = Factor of safety x shape factor.

(vii) Factor of safety:

(a) F.O.S. for ductile materials like mild steel

$$= \frac{\text{yield stress}}{\text{working stress}}$$

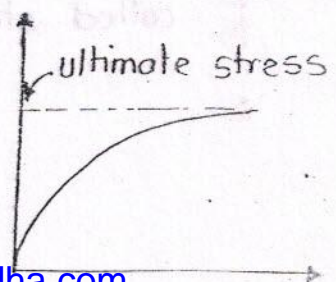
$$= \frac{f_y}{f}$$



(b) F.O.S. for brittle materials like concrete

$$= \frac{\text{ultimate stress}}{\text{working stress}}$$

(no clearly defined yield point)





(c) Margin of safety (M.O.S.)

(valid only for brittle materials)

$$\text{Margin of safety} = F.O.S. - 1$$

$$= \frac{\text{ultimate stress}}{\text{working stress}} - 1$$

$$= \frac{\text{ultimate load}}{\text{working load}} - 1$$

(viii) Moment-curvature relationship :

We know, Flexure formula,

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

where

R - radius of curvature of bent up beam.

$\frac{1}{R}$  - curvature.

y - deflection (deviation of beam from the initial configuration)

$\theta = \frac{dy}{dx}$  - slope (rate of change of deflection along the length)

$\frac{1}{R} = \frac{d^2y}{dx^2} = \frac{d\theta}{dx}$  - curvature (rate of change of slope along the length)

$$\frac{1}{R} = \frac{M}{EI}$$

curvature  $\propto M$

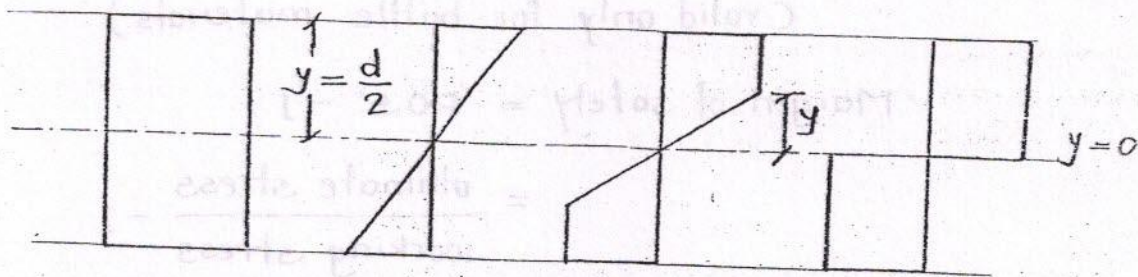
so as moment increases, curvature also increases.

(a) At yielding,

$$\frac{1}{R} = \frac{M_y}{EI}$$



\* (b) At fully plastic state, curvature in infinity.



$$\frac{f}{y} = \frac{E}{R}$$

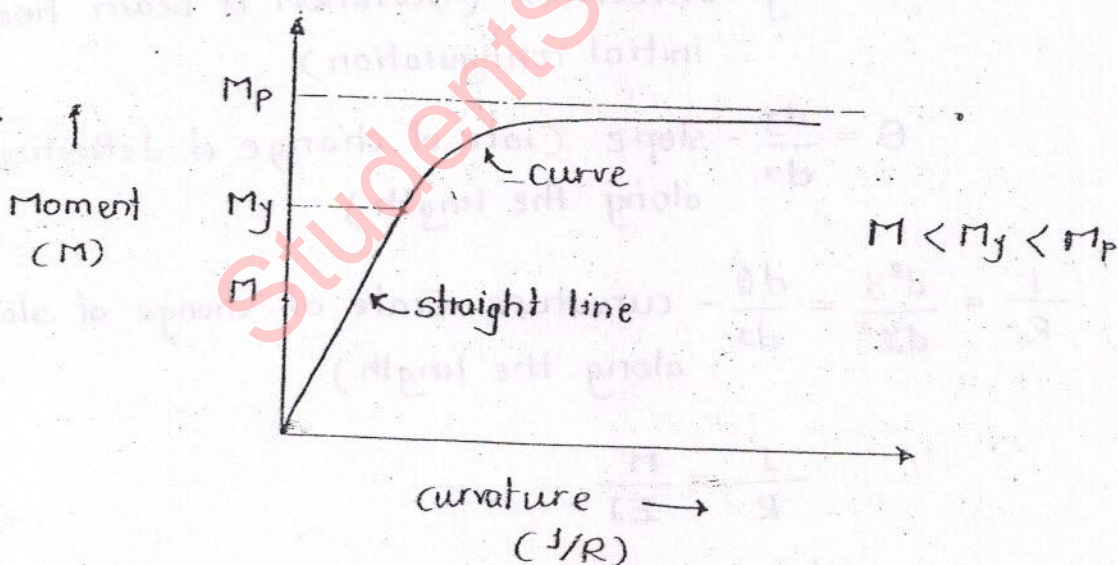
$$\frac{J}{R} = \frac{f}{y \times E}$$

where

$y$  - distance from N.A. to extreme fibre which is within proportionality limit.

$y = 0$  at fully plastic state

$$\frac{1}{R} = \frac{f}{0} = \infty$$



Moment-curvature relationship (based on idealised stress-strain curve)



Note:

$$M_p = f_y \cdot Z_p$$

$$= f_y \cdot \frac{A}{2} (\bar{y}_1 + \bar{y}_2)$$

The plastic moment carrying capacity of section depends on  $f_y$ , c/s area and the distribution of area (distribute the area in such a way that  $\bar{y}_1$  and  $\bar{y}_2$  are maximum so that  $M_p$  will be maximum)

Illustration:

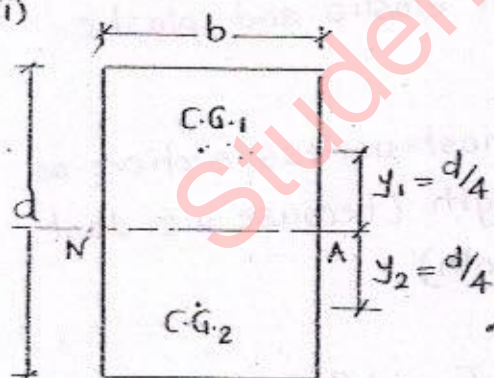
$$\text{shape factor} = \frac{M_p}{M_y} = \frac{5 \text{ kN}\cdot\text{m}}{2.5} = 2$$

$$\text{for another beam section, S.F.} = \frac{112 \text{ kNm}}{100 \text{ kNm}} = 1.12$$

Thus, S.F. is not measure of moment carrying capacity but just the ratio defining reserved moment carrying capacity.

Shear factors for different types of shapes: (c/s):

(i)



$$I = \frac{bd^3}{12}$$

$$Z_p = \frac{bd^2}{4}$$

$$\therefore \text{S.F.} = \frac{Z_p}{Z}$$

$$= \frac{\frac{A}{2} (\bar{y}_1 + \bar{y}_2)}{\left(\frac{bd^3}{12}\right)}$$

$$= \left(\frac{bd}{2}\right) \cdot (\bar{y}_1 + \bar{y}_2)$$

$$= \left(\frac{bd}{2}\right) \cdot (\bar{y}_1 + \bar{y}_2)$$

$$= \frac{bd^2/4}{bd^3/12}$$

$$= \frac{bd^2/4}{bd^3/12}$$

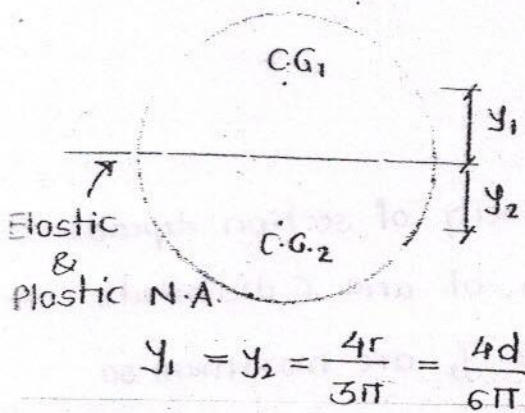
$$= 1.5$$

$$\therefore \bar{y}_1 = \bar{y}_2 = \frac{d}{4}$$

shape factor of 1.5 means, there is 50% of reserved strength beyond yielding (Reserved strength = 1.5  $M_y$ )



(ii) Solid circular section:



$$I = \frac{\pi d^3}{32}$$

$$Z_p = \frac{d^3}{6}$$

$$S.F. = \frac{Z_p}{Z}$$

$$= \frac{\frac{A}{2} (\bar{y}_1 + \bar{y}_2)}{\frac{\pi d^3}{32}}$$

$$= \frac{\left(\frac{\pi d^2/4}{2}\right) \cdot \left(\frac{4d}{6\pi} + \frac{4d}{6\pi}\right)}{\left(\frac{\pi d^3}{32}\right)}$$

$$= \frac{\left(\frac{\pi d^2}{8}\right) \cdot \left(\frac{4d}{3\pi}\right)}{\left(\frac{\pi d^3}{32}\right)} = \frac{\left(\frac{d^3}{6}\right)}{\left(\frac{\pi d^3}{32}\right)}$$

$$= \frac{32}{6\pi}$$

$$= 1.7$$

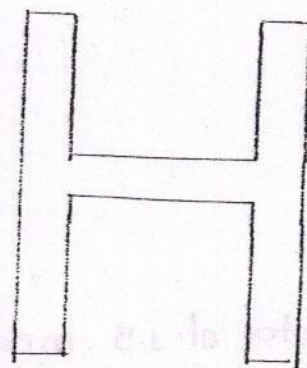
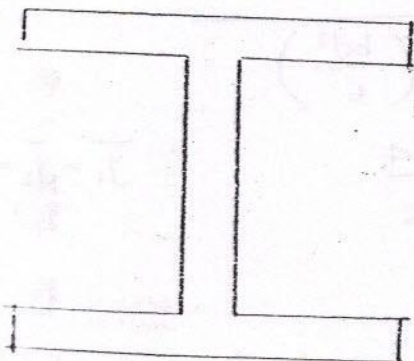
Note:

- (i) Elastic N.A. (from "strength of materials") passes through C.G. of cross section. Plastic N.A. cuts the entire area into two equal areas. So if the c/s is symmetrical about X-axis, then elastic and plastic N.A. coincide.
- (ii) The circular section will be most useless section as it has 70% of reserved strength (because we don't load section beyond yield strength)

(ii)

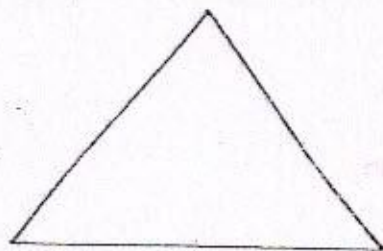
$$S.F. = 1.12 \text{ to } 1.14$$

$$(iv) S.F. = 1.5$$

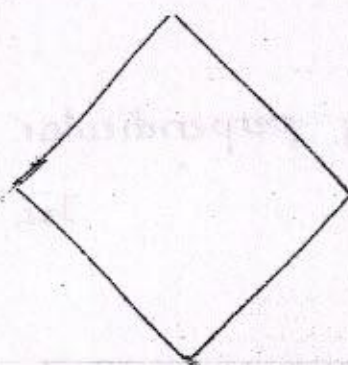




(v) S.F. = 2.34.

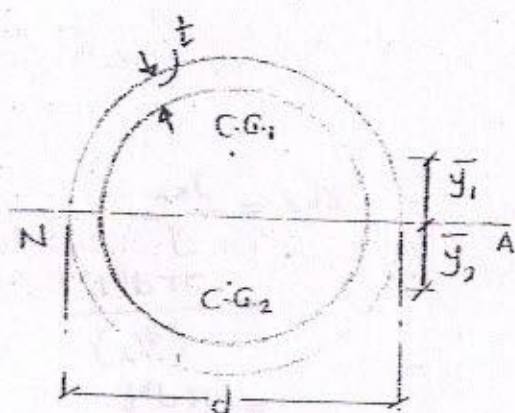


(vi) S.F. = 2



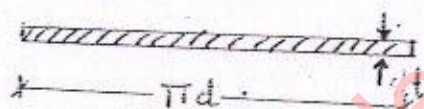
10 Marks

(vii) S.F. for thin hollow circular section



("Thin" implies that it is line area so  $t^2$  and  $t^3$  terms are neglected in calculations)

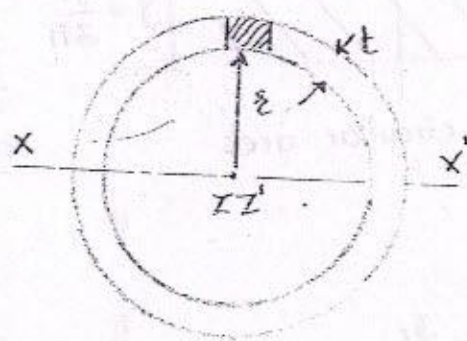
$$\begin{aligned} \therefore S.F. &= \frac{Z_P}{Z} \\ &= \frac{\frac{A}{2} (\bar{y}_1 + \bar{y}_2)}{Z} \end{aligned}$$



$\therefore A = \pi \cdot d \cdot t$

$y_1 = y_2 = \frac{2r}{\pi} = \frac{d}{\pi}$

$$\begin{aligned} Z_P &= \frac{\pi d t}{2} \left( \frac{d}{\pi} + \frac{d}{\pi} \right) \\ &= d^2 t \end{aligned}$$



$\therefore$  For calculating Moment of inertia, the elemental area should be at equal distance from axis

(for X-X' - elemental areas are at different distances)

$\therefore$  Take moment about Z-Z' axis which is perpendicular to X-X'

$$\begin{aligned} I_{zz} &= \int r \cdot dA && \text{(1st moment) of area} \\ &= \int r^2 \cdot dA && \text{(2nd moment or moment of inertia)} \end{aligned}$$

$I_{zz} = A \cdot r^2$



$$I_{zz} = \pi \cdot dt \left( \frac{d}{2} \right)^2$$

$$= \frac{\pi d^3 t}{4}$$

By perpendicular axis theorem,

$$J_{zz} = J_{xx} + J_{yy}$$

$$= J_{xx} + J_{xx} \quad (\text{because of symmetry})$$

$$J_{xx} = \frac{J_{zz}}{2}$$

$$= \frac{\pi d^3 t}{8}$$

$$\therefore \text{shape factor, } S.F. = \frac{Z_p}{Z}$$

$$= \frac{d^2 t}{\left( \frac{\pi d^3 t}{4} \right)}$$

$$= \frac{4}{\pi}$$

$$= 1.27$$

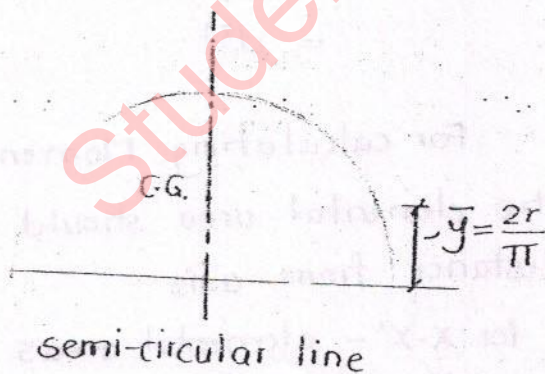
$$\therefore Z_{xx} = \frac{J_{xx}}{y}$$

$$= \frac{\frac{\pi d^3 t}{8}}{\left( \frac{d}{2} \right)}$$

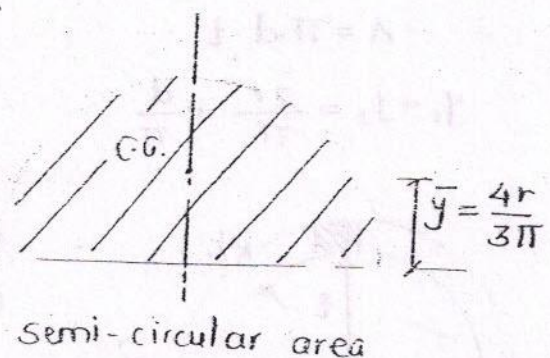
$$= \frac{\pi d^2 t}{4}$$

Note:

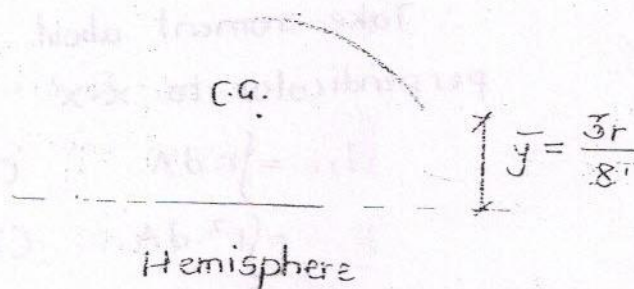
(i)



Semi-circular line



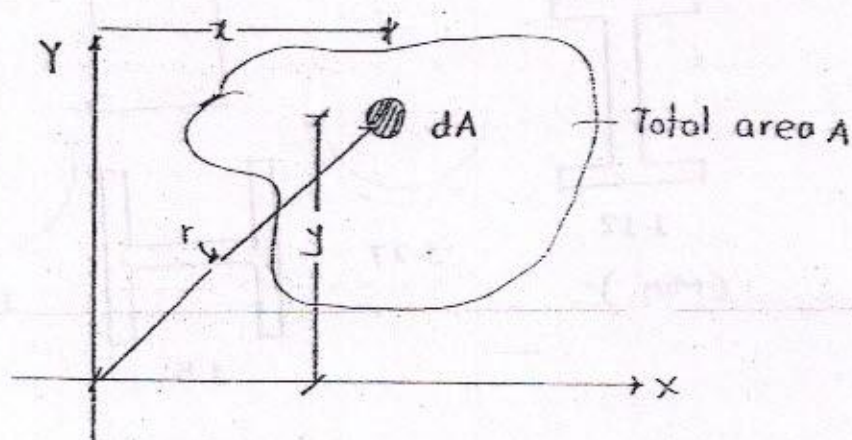
Semi-circular area



Hemisphere



(ii) Second moment of area or moment of inertia:



$y \cdot dA$  is called first moment of area.

$y^2 \cdot dA$  is called second moment of area or moment of inertia.

$$I_{xx} = M.I. = \int y^2 \cdot dA$$

$$I_{yy} = \int x^2 \cdot dA$$

$$I_{zz} = \int r^2 \cdot dA$$

$$= \int (x^2 + y^2) \cdot dA$$

$$I_{zz} = I_{xx} + I_{yy}$$

The term  $y^2 \cdot dA$  (Moment of inertia) combined with modulus of Elasticity (Material property) gives the measure of resistance to rotation or buckling (i.e. rigidity)

e.g. (i) Crippling load  $= P_c = \frac{\pi^2 EI}{L^2}$  (for column)

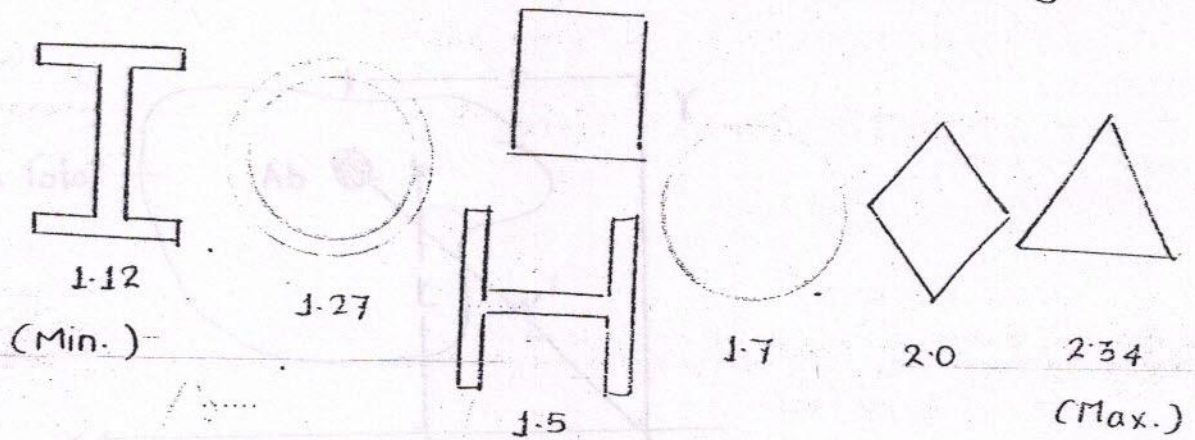
More  $EI$ , more is the rigidity against the buckling.

(ii)  $M = \frac{f}{y} \times I$

More value of  $I$  means more rigidity against bending.



(iii) Ascending order of shear factor: (increasing)



Q. For a rectangular section, shape factor is 1.5, permissible bending stress is  $0.66 f_y$ . Load factor is —

$$\text{Load factor} = (\text{F.O.S.}) \times \text{S.F.}$$

$$= \left( \frac{f_y}{f} \right) \times 1.5$$

$$= \left( \frac{f_y}{0.66 f_y} \right) \times 1.5$$

$$= 1.5 \times 1.5$$

$$= 2.25$$

$$\text{F.O.S.} = \frac{f_y}{f}$$

$$f = 0.66 f_y$$

$$\sigma_{cc} = \frac{f_{ck}}{4}$$

$$\sigma_{cbc} = \frac{f_{ck}}{3}$$

Q. For an I-section S.F. is 1.12. F.O.S. in bending is 1.5. The allowable is increased by 20%. then the load factor, is —

$$\text{Load factor} = \text{F.O.S.} \times \text{S.F.}$$

$$= \left( \frac{f_y}{f} \right) \times 1.12$$

If  $f$  increased by 20%

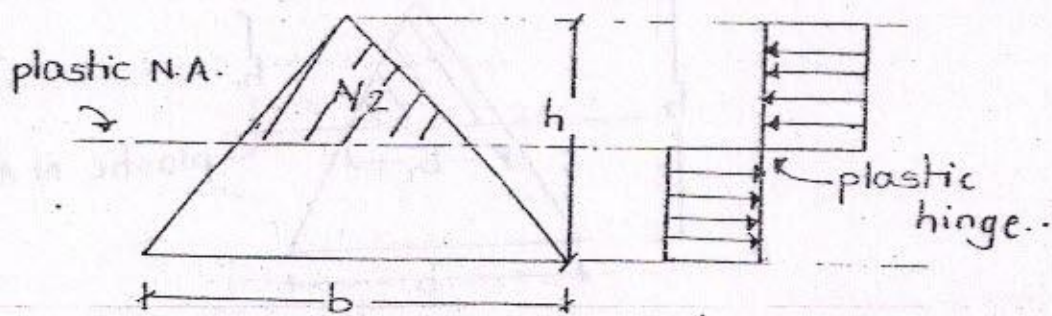
$$\text{L.F.} = \left( \frac{f_y}{1.2 f} \right) \times 1.12$$

$$= \frac{1}{1.2} \times 1.12 \times \frac{1}{0.66}$$

$$= 1.4$$



Q. When a triangular section of a beam becomes a plastic hinge then compressive force acting on section is

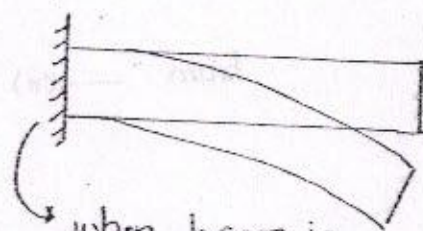


$$\begin{aligned}
 C &= f_y \cdot \frac{A}{2} && \text{At plastic state} \\
 &= f_y \times \frac{1}{2} \times (b \times h) \times \frac{1}{2} \\
 &= \frac{bh}{4} \cdot f_y
 \end{aligned}$$

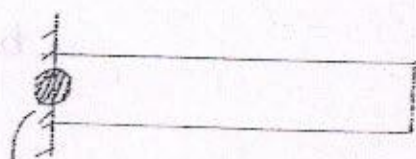
Note:

- Whatever may be the shape of c/s, stress distribution is rectangular at plastic state.
- If the cross section of beam is subjected to  $f_y$  throughout its depth, then it cannot take any further loading. If any additional load is applied then the beam rotates at that c/s. Since it rotates due to the moment  $M_p$ , we say that a plastic hinge is formed at that section.

Difference between hinge (provided in beams) and plastic hinge (formed in beam due to  $M_p$ )



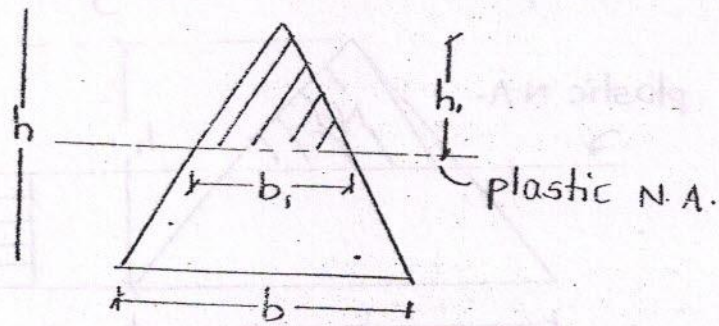
when beam is subjected to  $f_y$ , beam rotates (plastic hinge)  
(At  $M_p$ )



drilled and bolt is provided throughout  
(Analogous to hinge in beam)



In the above problem, depth of plastic N.A. from top is



No. of unknowns = 2 i.e.  $b, h$ .

No. of eqn required = 2

(i) Plastic N.A. cuts the given area into two equal parts.

compression area =  $\frac{1}{2} \times \text{total area}$

$$\frac{1}{2} \times b_1 \times h_1 = \frac{1}{2} \times b \times h$$

$$h_1 = \frac{bh}{2b_1} \quad \text{--- (i)}$$

(ii) From a similar triangle concepts.

$$h \rightarrow b$$

$$h_1 \rightarrow b_1 = ?$$

$$b_1 = \frac{h_1}{h} \times b$$

$\therefore$  from --- (i)

$$h_1 = \frac{bh}{2 \times \frac{h_1}{h} \times b}$$

$$h_1^2 = \frac{h^2}{2}$$

$$h_1 = \frac{h}{\sqrt{2}}$$

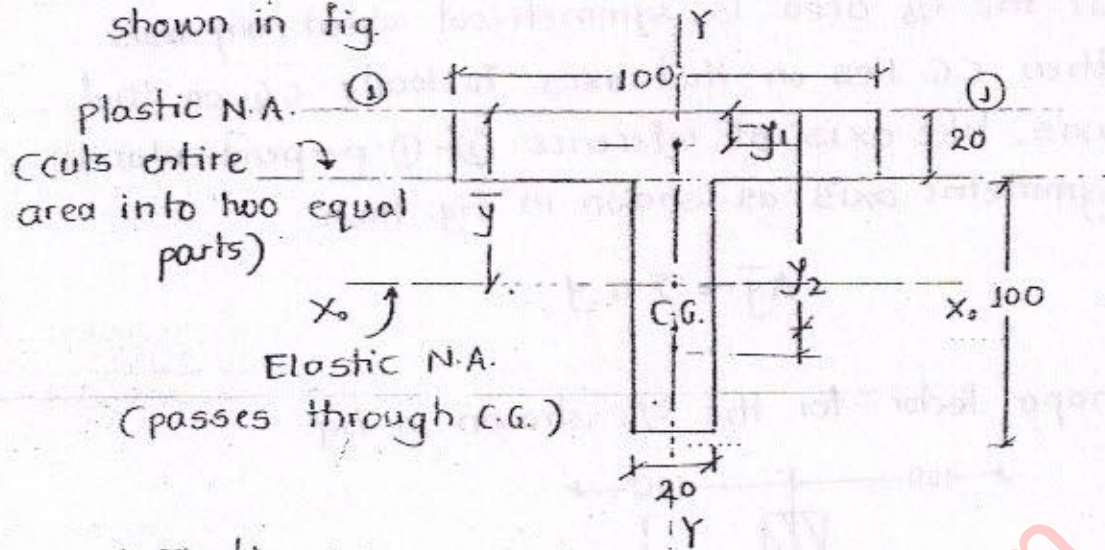
$\therefore$

$$b_1 = \frac{b}{\sqrt{2}}$$

from --- (ii)



Q. The distance between elastic and plastic N.A. for section shown in fig.



(i) Elastic N.A.

From Varignon's theorem (i.e. Moment of total area is equal to moment of summation of component areas)

i.e.

$$A \times \bar{y} = a_1 y_1 + a_2 y_2 + \dots = \sum a y$$

$$\bar{y} = \frac{\sum a y}{A}$$

$$= \frac{[(100 \times 20) \times 10] + [(100 \times 20) \times 70]}{(100 \times 20) + (100 \times 20)}$$

$$= 40 \text{ mm} \quad (\text{location of elastic N.A. from top})$$

(ii) Plastic N.A.

Since the web area and Flange area are equal plastic N.A. passes through junction of Flange and web i.e. at distance of 20 mm from top.

(a) Distance between plastic and elastic N.A.

$$= 40 - 20$$

$$= 20 \text{ mm}$$

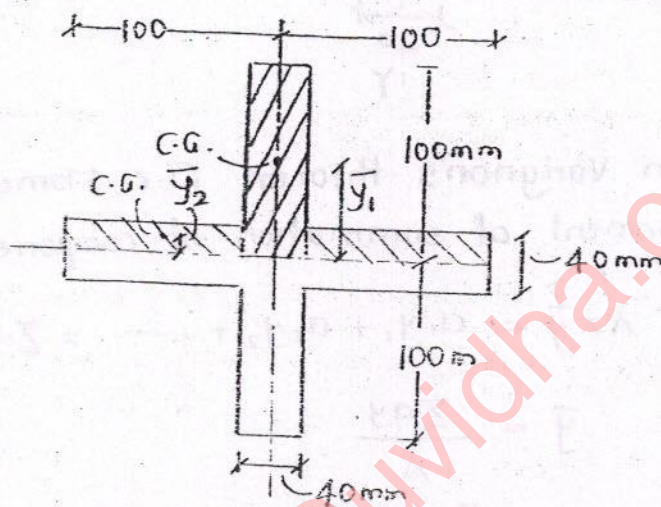


Note:

- (i) If the c/s area is symmetrical about any axis then C.G. lies on that axis. To locate C.G. on that axis, take axis of reference ①-① perpendicular to symmetric axis as shown in fig. then

$$A\bar{y} = \sum a \cdot y$$

Q. Find the shape factor for the c/s shown in fig.



$$\therefore \text{shape factor} = \frac{Z_p}{Z}$$

To find  $Z_p$

$$Z_p = \frac{A}{2} (\bar{y}_1 + \bar{y}_2)$$

$$= (40 \times 100) (50 + 50) + (160 \times 20) (10 + 10)$$

$$= 400000 + 64000$$

$$Z_p = 464000 \text{ mm}^3$$

To find  $Z$

$$Z = \frac{I_{xx} \text{ of entire c/s}}{y}$$

$$= \left[ \frac{40 \times 200^3}{12} \right] + \left[ \frac{160 \times 40^3}{12} \right] \quad \left/ \quad 100 \right.$$

vertical rectangle

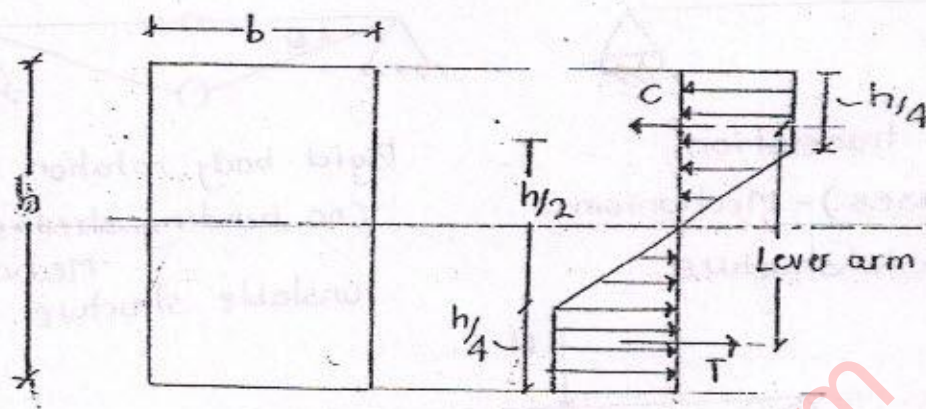
horizontal rectangle

$$= \frac{40 \times 200^3 + 160 \times 40^3}{1200}$$

$$\therefore \text{S.F.} = \frac{Z_p}{Z} = \frac{145}{80}$$



Q. A rectangular section of width 'b' and depth 'h' has been stressed upto a depth  $\frac{h}{4}$  upto  $f_y$  from both top & bottom fibres under the action of moment M. The magnitude of M is —



Total  $M = M_1 + M_2$   
 B.M. due to triangular stress distribution      B.M. due rectangular stress distribution.

$$= (f_y \cdot Z) + (C \times \text{lever arm})$$

$$= \left( f_y \times \frac{b \times \left(\frac{h}{2}\right)^2}{6} \right) + \left[ \left( f_y \times \frac{b \cdot h}{4} \right) \times \left( \frac{3h}{4} \right) \right]$$

$$= \frac{f_y \times b h^2}{24} + \left[ f_y \times \frac{b h^2 \times 3}{16} \right]$$

$$M = \frac{11 b h^2}{48} f_y$$